

Exercise 78

Find the 1000th derivative of $f(x) = xe^{-x}$.

Solution

Differentiate the function several times using the product and chain rules and see if a pattern emerges.

$$y^{(1)} = \frac{d}{dx}(xe^{-x}) = (1-x)e^{-x}$$

$$y^{(2)} = \frac{d}{dx}[(1-x)e^{-x}] = -(2-x)e^{-x}$$

$$y^{(3)} = \frac{d}{dx}[-(2-x)e^{-x}] = (3-x)e^{-x}$$

$$y^{(4)} = \frac{d}{dx}[(3-x)e^{-x}] = -(4-x)e^{-x}$$

It appears that even derivatives are negative.

$$y^{(1000)} = -(1000-x)e^{-x}$$